UNIT= Ist

NOISE , VIBRATION & ~~SOUND~~

Sound:- Sound is form of energy transmitted from a vibration source. The vibrating matter creats small repetitive presure distribution that are Important to the air along a path and reach a reciever, the ear.

A wave is a distrubance that propagate through space ar space time after transferring energy.

A mechemical wave exits in a medium.

Subjactives response to sound:-

Subjective response is directly measured by human Ideaar opinion.

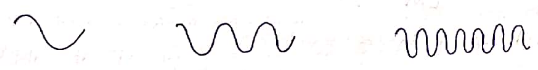
The subjective response is key factor in noise contral becaun all the complaints are expected from human being who may subjected of loss of sleep.

Noise genreution is associated with most of our daily activit A healthy human ear ~~repo~~ responds to a very wide range of Sound presure level (SPL) - the threshald of hearing at zero dB uncomfortable at 100-1200 dB and painful at 130-140 dB.

Due to vevioUs Impact of noise on human and environmen noise should be contralled.

Frequency dependent human response to sound:-

Noise consist of regularly repeated as periodic sound and those that consist of a preiodic sound. The simplest periodic is a pure tone i.e. a presure distribution the fluctuates sinusoidally at a particular frequency. The lower the frequency the longer is the wavelength.



A normal human ear is also able to hear sound frequencies from 20Hz to 20,000 Hz. This range also called the audiable frequency range. The sound we hev hear comprise of various frequencies. The entire auidable frequency rang can be divided into 8 or 24 frequency bands known as octave band or 1/3 octave band for analysis. The sound in the frequency range 15000 Hz to 20,000Hz may be audible to there people heaving sharp heaving.

The sound at frequencies above 20 K Hz is therefore called ultrasonic sound. whereas on the lower frequency side the ~~audi~~ audible frequency rang extends up to 15Hz. can not be heared but can be felt only if they are sufficiently strong such sound are called Infrasonic sound. Infrasonic occurs both naturally and ~~art~~ artificially.

⇒ Where there is no ambiant sound in a room the lowest sound Intensity that can be barely detected at any particular frequency known as the threshold of heaving or threshold of audibrity at that frequency.

Sound Pressur depenedent human response:-

Quantification of sound:-

1. Sound power level:-

Sound power is the rate at which acoustic energy is ~~transm~~ radiated from a sound source. It can be expressed in watt or decibels dB

Sound power level (PWL) is defined as the logarithmic ratio of the sound power emitted to refrence sound power.

$$L_W = 10 \log_{10} \frac{\text{Sound Power}}{\text{Reference sound power}}$$

$$= 10 \log_{10} \frac{W}{W_0} \begin{array}{l} \rightarrow \text{sound power measured in watts} \\ \rightarrow \text{Refrence sound power} = 10^{-12}\,W \end{array}$$

Internationally agreed refrence power is 10^{-12} W

$$\boxed{L_W = 10 \log_{10} W + 120\,dB}$$

2. Sound Intensity level:- Sound Intensity is a vector quantity.

$$\text{Intensity} = \frac{\text{Energy}}{\text{Time} \times \text{Area}} = \frac{\text{Power}}{\text{Area}}$$

The Sound Intensity is related to the root mean square (rms) acoustic premur as follows.

$$\boxed{I = \frac{P^2_{rms}}{\rho c}}$$

$\rho \rightarrow$ Density of air in kg/m³

C = Speed of sound in m/s = $\sqrt{\gamma RT}$

ρc – Acoustic Impedance 414 NS/m³ at STP condⁿ.

A sound Intensity level L_I may be define as follows:

$$L_I = 10 \log_{10} \frac{\text{Sound Intensity}}{\text{Reference sound Intensity}} = \frac{I}{I_{ref}}.$$

$$L_I = 10 \log_{10} I + 120\ (dB)$$

Sound Pressure level:- The range of sound pressure that can be heard by the human ear is very large. The minimum acoustic pressure audible to the young human ear judged to be in good health. it approximately 2×10^{-6} Pa or 2×10^{-10} Pa atmospheres.

⇒ The decibel is a logarithmic expression of power ratio. Sound pressure is proportional to field. sound pressure squared and the sound pressure level is therefore:

$$L_p = 10\log_{10}\left(\frac{P^2_{rms}}{P^2}\right)_{ref} \text{ (dB)}$$

The rms value of sound pressure at 140dB is only 200Pa.

$$L_p = 20\log_{10}\left(\frac{P_{rms}}{P}\right)_{ref} \text{ (dB)}$$

P_{rms} = Root mean square pressure

$\cdot \frac{P}{\sqrt{2}} = 0.707P$

$P_{ref} = 2\times10^{-5}$ Pa or 20μPa

$$\boxed{L_p = 20\log_{10} P_{rms} + 94 \text{ (dB)}}$$

Noise:-

Noise is perceived by the ear as fluctuation in air pressure. The rate at which these fluctuations changes determine the frequency of the sound and the pressure generated determine the intensity or sound lev

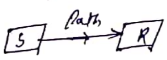
Effects of Noise:-

Noise induce a severe impact on human and on living organism. so of the adverse effects are given below:-

- **Annoyance:-** The aperiodic sound due to its irregular occurrences cau displeasure to hearing and causes annoyance.
- **Physiological effects:-** It's features like breathing amplitude, blood pre heart beat rate, pulse rate, blood cholesterol are affected.
- **Loss of hearing:-** Long exposure to high sound level causes loss of heari This mostly unnoticed but has an adverse impact on hearing functi
- **Human Performance:-** The working performance of worker/human will be affected as they will be loosing their concentration.
- **Damage to material:-** The buildings and materials may get damaged by ~~expouse~~ exposure to infrasonic / ultrasonic waves and even ge collapsed.

Inter-relationship between the element of Noise:-

A noise problem generally related three elements:- Source, the receiver & the transmission path. This transmissi path is usually the atm through which the sound is propogat



Non- auditory effects of noise on peoples:-

Non- auditory effect of noise refer to impacts not directly related to sound damage to the auditory system. Non auditory effects of noise such as elevated sympathetic nervous system activity or disturbances in attentional process.

→ Non auditory effects like hearing loss, stress tension and adverse health effects. It also causes stress or tension due to their physio-psychological impact.

Effects of Non- auditory:-

1 An increase in the number of peopele with psychological and physiological health problems requiring the increased use in certain type of drugs and visit to physicians.

2 An increase in the incidence of female infants with reduced gestation period and body weight at the time of birth.

3 An increase in the number of adults requiring admission of psychiatri hospitals.

(1) Periodic motion:-

A motion which repeat it self after equal interval of time.

2 Time Periode:-

Time taken to complet one cycle

3 frequency:-

No. of cycles per unit time

$f = \frac{1}{T} = 1/_{sec} = Hz$ or $\omega_n = \frac{rad}{sec}$

$\omega = 2\pi f$

~~f~~= $\omega = \frac{2\pi}{T}$

4 Amplitude:-

The maximum displacement of a vaibrating body from its equilibrium position.

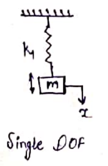
5 Natural frequency:-

The vibration in which there is no kinematic friction as well as there is no external force. after the initial release of the system are known as natural vibration.

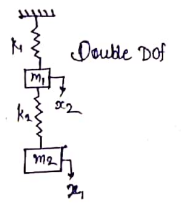
Ex. Simple pendulum.

6 Degree of freedom (DoF):-

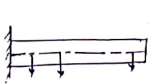
minimum no. of independent co-ordinate required to specify position / motion of a system at any instant is known as degree of freedom.



Single DOF



Double DoF



Simple Harmonic Motion:-

The motion of a body To and fro about a fixed point is called SHM.

- The motion is periodic.
- It's acceleration is always directed toward the mean position and is perpotional to its distance from mean position.

$x = A\sin\omega t$ ——①

$\dot{x} = A\omega\cos\omega t$ ——②

$\ddot{x} = -\omega^2 x$ ——③

Damping:-

It is the resistance the motion of a vaibrating body, the vibration associated with this resistance are known as demped vibrations.

Phase Difference:-

Suppose there are two vector x_1 and x_2 having frequency ω(rad/sec) each the vibrating motion can be expressed as $x_1 = A_1 \sin\omega t$

$x_2 = A_2 \sin(\omega t + \phi)$

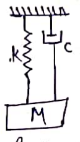
ϕ is known as phase difference.

Resonance:-

When the frequency of external excitation is equa to natural frequency of a vibrating body. the amplitude of vibrati become excessively large. this concept is known as resonance.

mechanical System:-

The system consisting of spring mass and damper



Spring - K

Damper - C

mass = M

$$\boxed{m\ddot{x} + c\dot{x} + kx = 0}$$

Parts of Vibrating system

*Contineous & Discrate System:- most of the mechanical system include elastic members which have infinite number of degree of freedom (Centilever beam, simply supported beam etc) such system is called Contineous or discrate system.

System with finite no. of dof are called discrate or lumped System.

Reasons of vibration & method to Remove the vibration:-

⇒ Unbalanced c.f.
→ External excitation
→ Earthquake
→ wind causes vibration in Telephone & electricity cables.
→ Dry friction
→ Elastic nature of the system.

Romove the vibration:-

→ By proper balancing.
→ By proper lubrication
→ By remove external excitation.
→ By using shock absorbers.
→ By resting the system on proper foundation.

Types of vibration:-

- Free and forced vibration:-

After disturbing the system the external excitation is remove then the system vibrate on its on. (Simple pendulam)

The vibration which is under the influence of external excitation (force), is called forced vibration.

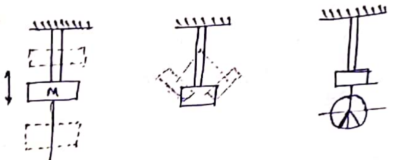
Ex: electric bell, Horn.

- Damped and undamped vibration:-

If the vibatery system has a damper the motion of the system will be apposed by it and the energy of the system will be dissipited in friction

On the contrary the system having no damper is known as undamped vaibration.

- Longitudnal, Transverse & Torsional:-



1 If mass M moves up and down parllel to the spindle axis are know as longitudnal

2 If partical of body or shaft move approximately perpendicular to the axis of shaft.

3. If the spindle gets alternately twisted and untwisted on account of vibaratory motion of the suspended ~~disk~~ disc.

4 linear and non linear vibration:-

If in a vibratory system spring mass and damper behave in a linear manner.

If any of the basic component of a vibrating system behave non linearly.

linear vibration becoms non linear for very large amplitude of vibration.

5 Deterministic & Random:- If in the vibratory system amount of external excitation is known deterministic.

Contanary to it the non determinsitic vibration are known as Random vibration.

Transient vibration:-

In ideal system the free vibration contineous indefinately has their is no damping. but the amplitude of vibration decays contineously becoz of demping. (In a real system). and vanishes ultimeately such vibration in a real system is called transient vibration.

First Question is Compulsory and out of remaining

* SHM , Addition of SHM , complex number Representation & Vectorial Representation:-

Let $x_1 = A_1 \sin\omega t$, $x_2 = A_2 \sin(\omega t + \phi)$

$x = x_1 + x_2$

$x = A_1 \sin\omega t + A_2 \sin(\omega t + \phi)$

$x = A_1 \sin\omega t + A_2 \sin\omega t \cos\phi + A_2 \sin\phi \cos\omega t$

$x = \sin\omega t\,(A_1 + A_2\cos\phi) + A_2 \sin\phi \cos\omega t$ —— ①

Assume:-

$A_1 + A_2 \cos\phi = A\cos\theta$ —— ⓐ

$A_2 \sin\phi = A\sin\theta$ —— ⓑ

$x = A \sin\omega t \cos\theta + A\sin\theta \cos\omega t$

$\boxed{x = A\sin(\omega t + \theta)}$ —— ② θ - Resultant phase difference
A - Resultant Amplitude

Equation ② is also represent of ~~of~~ SHM

$$\boxed{\theta = \tan^{-1}\left(\frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}\right)}$$

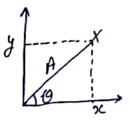
$$\boxed{A = \sqrt{A_1^2 + \cancel{A_2^2} + 2A_1A_2\cos\phi}}$$

$$\boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}}$$

*Complex Number:-

Let a Vector $X = \underset{\text{Real part of the Vector}}{x} + \overset{}{iy} \leftarrow$ Imaginary part

$x = A\cos\theta$
$y = A\sin\theta$
$\boxed{X = A\cos\theta + i\,A\sin\theta}$



$\boxed{X = Ae^{i\omega t}}$ (Euler formula)

$\dot{X} = Ai\omega e^{i\omega t}$

$\ddot{X} = -A\omega^2 e^{i\omega t}$

$\boxed{\ddot{X} = -\omega^2 X}$

Vectorial Representation:-

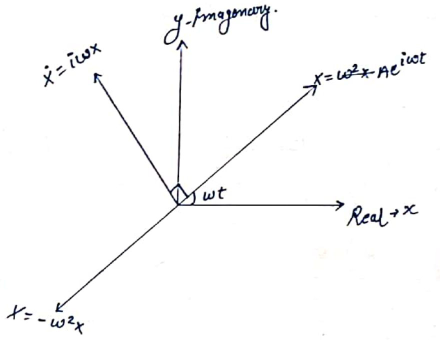
$x = A\sin\omega t$

$\dot{x} = A\omega\cos\omega t$

$\dot{x} = A\omega\sin\left(\omega t + \frac{\pi}{2}\right)$

$\ddot{x} = A\omega^2\cos(\omega t + \pi/2)$

$= A\omega^2\sin(\omega t + \pi)$



Q: Add the following harmonic motions analytically & check the solution graphically?

Sol:

$x_1 = 4\cos(\omega t + 10°)$

$x_2 = 6\sin(\omega t + 60°)$

$x_1 = 4\sin(\omega t + 100°)$

$x_2 = 6\sin(\omega t + 60°)$

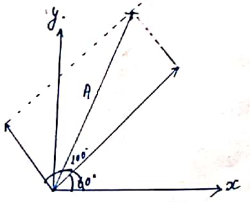
$x = x_1 + x_2$

$= 4\sin(\omega t + 100) + 6\sin(\omega t + 60°)$

$= (4\sin\omega t\cos 100 + 4\sin 100\cos\omega t) + (6\sin\omega t\cos 60° + 6\sin 60\cos\omega t)$

$= -0.69\sin\omega t + 3.93\cos\omega t + 3\sin\omega t + 5.1\cos\omega t$

$\boxed{x = 2.31\sin\omega t + 9.03\cos\omega t}$

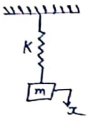


First Question is compulsory and out of remaining

Energy method:- In this method the total energy is constant "total energy is P.E & K.E for spring mass system.

$K.E = \frac{1}{2}m\dot{x}^2$

$P.E = \frac{1}{2}\cancel{m}kx^2$



Total Energy = constant

K.E + P.E = constant

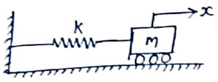
$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = 0$

Diffrenciation with respect to time.

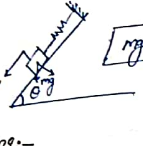
$2\times\frac{1}{2}m\dot{x}\ddot{x} + \frac{1}{2}\times2\times k\,x\dot{x} = 0$

$\boxed{m\ddot{x} + kx = 0}$

Spring mass system in Horizontal



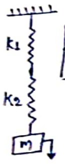
Spring mass system in Inclined



$\boxed{mg\sin\theta = K\delta}$

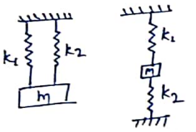
Equivalent stiffnes of spring & Cutting of spring:-

Series



$\boxed{\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \cdots}$

Parllel

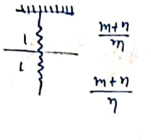


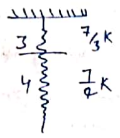
$\boxed{K = K_1 + K_2 + \cdots}$

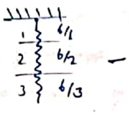
Cutting of Spring:-

Stiffness ∝ $\frac{1}{\text{leglength}}$

Stiffnes ∝ Area







Solution of equation of motion of undamped single DOF:-

$\boxed{m\ddot{x} + kx = 0}$

This is the second order diffrential Equation.

$\boxed{\ddot{x} + \frac{k}{m}x = 0}$

$\boxed{x = R\sin\left(\sqrt{\frac{k}{m}}\times t + \phi\right)}$

Where: R, φ are two constant.

The final Solution:-

$\ddot{x} + \omega_n^2 x = 0$ ——— ②

$\omega_n^2 = \frac{K}{m} \Rightarrow \omega_n = \sqrt{\frac{K}{m}}$, $\boxed{f_n = \frac{\omega_n}{2\pi}}$, $\boxed{T = \frac{2\pi}{\omega_n}}$

Q What is the natural frequency of this:

I step make of all in Equilant manner

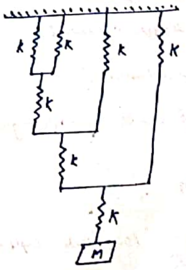
M = 20kg

K = 2×10⁵

then ω_n = ?

from Diagram:-

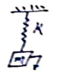
$\boxed{W_n = 78.67Hz}$



~~Tosional Vibrations~~-

Note- In spring mass system if mass of spring is considered then the natural frequency of spring mass system is given by.

$$\boxed{\omega_n = \sqrt{\frac{k}{m+\frac{m_s}{3}}}}$$



Natural frequency of compound pendulum

$$\omega_n = \sqrt{\frac{mgh}{I}} = \sqrt{\frac{mgh}{mk^2+mh^2}}$$

$$\boxed{\omega_n = \sqrt{\frac{gh}{k^2+h^2}}} \quad \therefore \boxed{T = 2\pi\omega_n = 2\pi\sqrt{\frac{gh}{k^2+h^2}}}$$

Torsional Vibration:-

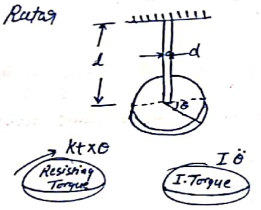
A rotor of mass moment of Inertia I is attached to a weightless shaft of dia d & stiffness K_t

I- mass moment of Inertia of Rotor

K_t- stiffness of shaft.

d- dia of shaft.

θ- angle of twist



Acc. D-Alembert

$$\boxed{K_t\theta + I\ddot{\theta} = 0} \rightarrow \{\text{Angular}$$

$$\boxed{\omega_n = \sqrt{\frac{K_t}{I}}}$$

from Torsional Equation

$$\frac{I}{J} = \frac{G\theta}{l}$$

$$\boxed{K_t = \frac{I}{\theta} = \frac{GJ}{l}}$$

J- Polar moment of Inertia of shaft

$$J = \frac{\pi}{34} \times d^4$$

l- length of the shaft.

* Moment of Inertia of some common objects:-

→ Ring & hollow Cylinder: $\frac{\pi}{32}$ $I = mR^2$ for disk / hollow cylinder.

→ Solid Disc & solid cylinder $I = mK^2 = \frac{1}{2}mR^2$

→ Hollow sphere :- $\frac{2}{3}mR^3$

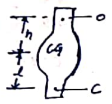
→ Solid sphere: $\frac{2}{5}mR^2$

} IMP

* Compound pendulum & center of percussion:-

Center of percussion:- The center of oscillation is some time ~~turn~~ termed as Center of percussion. defined as " the point which if the blow is struck on a suspended body there is no reaction as support.

where:-
- O → Point of suspension
- G → C.G
- C → Center of percussion.



*Concept of center of percussion:-

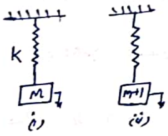
1 A hammer can ~~Sheared~~ Shaped to have the cop at the hammer ~~bed~~ head while the center of rotation at the handle the Impact force at the hammer head will not cause any normal reaction at the handle.

2. In base ball bat if on one hand the ball is made strick at the center of percussion while the center of rotation is at the hand no reaction perpendicular to the bat will be expriened by the batsmen.

If the ball struck near the hands the batsmens expriemced ~~pain~~ poin in the hand.

3 In an automobile if the front wheel strick a bump the passenger not feel any reaction if the center of percussion of the vehicle is located near the rear axle. if a rear wheel strick a bump no reaction will felt at the front axle. if the center copiylaadan near the front axle.

Q An unknown mass m is attached to one end of spring of Stiffnen k having natural frequency of 6Hz. When 1kg mass is attached with m is natural frequency of the system is lowered by 20%. determine the value of unknown mass m & k.



Ans:-

Given data:- $f_n = 6Hz$ K

If f_n = 20% lower when attached $M+1$

then $f_n = \frac{6 \times 80}{100} = 4.8 Hz$

then $\omega_n = \sqrt{\frac{K}{m}}$ $\omega_n = \sqrt{\frac{K}{m+1}}$

$f_n = 2\pi\sqrt{\frac{K}{m}}$. $f_n = 2\pi\sqrt{\frac{K}{m+1}}$

$\frac{6}{2\pi} = \sqrt{\frac{K}{m}}$, $\frac{4.8}{2\pi} = \sqrt{\frac{K}{m+1}}$ X

$0.95 = \sqrt{K/m}$; $0.71 = \sqrt{\frac{K}{m+1}}$

$0.90 = K/m$ ~~0.76~~ $0.57 = \frac{K}{m+1}$

$\boxed{K = 0.90m}$ —① $0.57m + 0.57 = K$ ——②

K $m = \frac{K - 0.57}{0.57} = 0.90$

$0.90m - 0.57m = 0.57$

$m = 1.77 kg.$ X

K = ~~1.55~~

$f_n = \frac{\omega_n}{2\pi} = \sqrt{K/m}$

$6 = \sqrt{K/m}$ —①

$4.8 = \sqrt{\frac{K}{m+1}}$ —②

$36 = K/m$ —Ⓐ

~~4.8~~ $\times 23.04 = \frac{K}{m+1}$ —Ⓑ.

$K = 36m.$

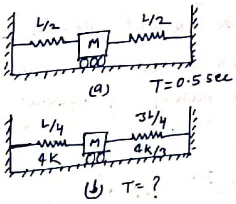
$23.04m + 23.04 = 36m$

$\boxed{m = 1.77 kg.}$

$\boxed{K = 64 N/m^2}$

solution

Num:- A helical spring of stiffnen of K is cut into two halves and a mass m is connected to the two halves as shown in fig. a.



for fig: a:-

$T = 2\pi\sqrt{\frac{m}{K}}$

$\frac{(0.5)^2}{4 \times (3.14)^2} = \frac{m}{4K} = 6.3 \times 10^{-3} \times 4$

$\boxed{M/K = 0.0252}$

for fig.

$\cancel{8} \times \frac{T}{4 \times 3.14^2} = \frac{m}{\cancel{8}K}$

fig (b) $T = 2\pi\sqrt{\frac{m \times 3}{16K}}$

$T = 2\pi\sqrt{\frac{3}{16} \times 0.0252}$

$\boxed{T = 0.43 sec}$

Q: A 5kg mass attached to lower end of the Spring whose upper end is fixed, vibrates with a natural period of 0.45 sec determine the natural period when a 2.5 kg mass is attached to the mid point of the Same Spring. with upper end and lower end is fixed.

$T = 2\pi\sqrt{\frac{m}{k}}$

$0.45 = 2 \times 3.14 \sqrt{\frac{5}{k}}$

$k = \frac{4 \times (3.14)^2 \times 5}{(0.45)^2}$

$\boxed{k = 973.78}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{m}{4k}}$

$\boxed{T = 0.15 \text{ sec}}$

Q The natural frequency of spring mass system is fund is to be 2Hz when an additional mass of 1kg is added to the original mass m the natural frequency is reduced to 1Hz find the stiff k & m.

$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$4 = \left(\frac{1}{2 \times 3.14}\right)^2 \times \frac{k}{m}$

$4 \times 4 \times \pi^2 = k/m = 157.91$

$f_n = \frac{1}{2\pi}\left(\frac{k}{m+1}\right)$

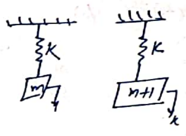
$39.43 = \frac{k}{m+1}$

$m =$

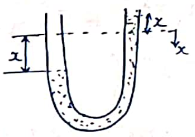
$k = 39.43m + 39.43$

$\boxed{m = 0.33 \text{ kg}}$ $\boxed{k = 52.11}$

Az



Q:- A simple U Tube menometer filled with liqued is shown in fig. Calculate the frequency of resulting motion if the minimum length of a menometer tube is 0.15 m.



Ans:: from Energy method:

$K.E + P.E = \text{Constant}$ — ①

$K.E = \frac{1}{2}mv^2, \quad P.E = \frac{1}{2}mx^2$

$K.E = \frac{1}{2}\rho A l \cdot \dot{x}^2, \quad$ ~~$P.E = \frac{1}{2}\rho \cdot A$~~

$P.E = mgh = \rho A \cdot x \times g \times x = \rho A g x^2$

then from equation ①

$\frac{1}{2}\rho A l^2 \cdot \dot{x}^2 + \rho A g x^2 = \text{Constant}$

Diff. w.r.t. time.

$\frac{\rho A l^2 \times 2\dot{x}}{2}\ddot{x} + 2\rho A g x \cdot \dot{x} = 0$

$\boxed{l\ddot{x} + 2gx = 0}$

$\ddot{x} + \frac{2g}{l}x = 0$

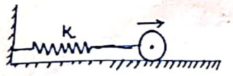
then $\boxed{\omega_n = \sqrt{\frac{2g}{l}}} = \sqrt{\frac{2 \times 9.81}{0.15}} =$

$f_n = \frac{1}{2\pi}\sqrt{\frac{2 \times 9.81}{0.15}}$

$\boxed{f_n = 1.82 \text{ Hz}}$

Ans

Q: A Circular cylinder of mass 4kg & radius 15cm is connected by a spring of stiffnes 4000N/m as shown in fig. It is free to roll on horizontal rough surface without slipping Determine the natural frequency.



Ans:- Apply Energy method.

K.E + P.E = Constant

$$K.E = \left(\frac{1}{2} m \dot{x}^2\right)_L + \left(\frac{1}{2} I \dot{\theta}^2\right)_{\text{Rotary motion}}$$

$$P.E = \frac{1}{2} m x^2$$

then

$$\frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} K x^2 = \text{Constant.}$$

$$\frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} K r^2 \theta^2 = \text{constant}$$

Diff. w.r.t. time.

$$\frac{1}{2} m r^2 2 \dot{\theta} \ddot{\theta} + 2 \times \frac{1}{2} I \dot{\theta} \ddot{\theta} + 2 \times \frac{1}{2} \times K r^2 \theta \dot{\theta} = 0$$

$$m r^2 \ddot{\theta} + I \ddot{\theta} + K r^2 \theta = 0$$

$$\ddot{\theta} (m r^2 + I) + k r^2 \theta = 0$$

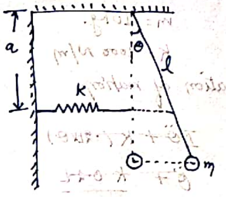
$$\ddot{\theta} + \frac{k r^2}{m r^2 + I} \theta = 0$$

$$\omega_n = \sqrt{\frac{k r^2}{m r^2 + I}} \quad \text{then:} \quad f_n = \frac{1}{2\pi} \sqrt{\frac{k r^2}{m r^2 + I}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{4000 \times (.15)^2}{4 \times (.15)^2 + \frac{1}{2} \times 4 \times (.15)^2}} = \frac{1}{2\pi} \sqrt{\frac{90}{0.135}}$$

$$\boxed{f_n = 4.11 \text{ Hz}}$$

Q:- Find the Equation of motion of there Diagram



Ans:- $I \ddot{\theta} + k\theta = 0$

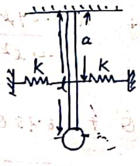
$I \ddot{\theta} + (Ka^2 + mgl)\theta = 0$ from D-Alembert

$$\ddot{\theta} + \left(\frac{Ka^2 + mgl}{ml^2}\right)\theta = 0$$

then $\boxed{\omega_n = \sqrt{\frac{ka^2 + mgl}{ml^2}}}$ Ans

Q Inertia force

$I\ddot{\theta}$ & Spring force



Spring force: ⇒

$(2ka^2 + mgl)\theta$

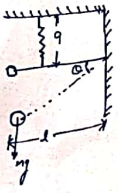
then from D-Alembert formula.

$$I\ddot{\theta} + (2ka^2 + mgl)\theta = 0$$

$$\ddot{\theta} + \left(\frac{2ka^2 + mgl}{ml^2}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{2ka^2 + mgl}{ml^2}}$$

Q $I\ddot{\theta}$ - Inertia force



Spring force $Ka\theta \times a$ - Spring torque

$mgl \times \cos\theta$ where $\theta = 0°$ then

mgl (there is no effect of θ)

So

Equation of motion

$$I\ddot{\theta} + Ka^2\theta = 0$$

$$\ddot{\theta} + \frac{Ka^2}{ml^2}\theta = 0$$

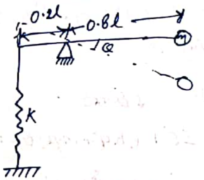
Then $\omega_n = \sqrt{\frac{Ka^2}{ml^2}}$

Q Given data

m = 10kg.

k = 1000 N/m

Equation of motion.



$~~I\ddot{\theta} + K(\cdot 2l \times \theta)~~$

$~~\ddot{\theta} + \frac{K\ 0.2L}{}~~$

$I\ddot{\theta} + (k\ 0.2l \times 0.2l \times \theta) = 0$

$I = ml^2 = 10 \times 0.8^2 l^2$

$= 6.4 l^2$

$\ddot{\theta} + \frac{K\ 0.04 l^2}{6.4 l^2}\ \theta = 0$

$\ddot{\theta} + \ldots 6.25\theta = 0$

$\omega_n = \sqrt{6.25}$

$\boxed{\omega_n = 2.5}$ natural frequencys.

Free undamped Vibration :-

- It also known as natural vibration. (free undamped vibration)
- free Damped vibration.

* Free damped vibration:-

⇒ Damped system:- (System having $\overset{No}{\text{}}$ rotatary part):-

Having No Rotary Part

- mass (kinematic Energy)
- Stiffness (Potential energy) → Due to position. (Elastic energy).
- kinematic friction
- fun ≠ 0

{ Vibration is the combination of 3-4 things / In Galilio Analysis $\boxed{K.F. = 0}$ → Natural frequency }.

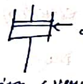
⇒ Factor on vibration depends:-

* But in real system kinematic friction is not equal to zero.

$\boxed{\text{Kinematic friction}}$:- Tech name in any ~~working~~ vibrating system is - Damping.

⇓

Energy loss

→ Represented by the symbol  ← Damper (Symbol & system)

Damping → (i) Due to dry friction. (very high) { Columb Damping }.

→ (ii) Viscous damping (Due to lubrication)

(Very less)

Viscous force is directly perpositional to the velocity of the System

$\boxed{\text{Viscous force} \propto \dot{x}}$

we know that

$\tau \propto \frac{du}{dy}, \quad \tau = \mu \frac{dy}{dy}$

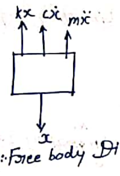
$\frac{F}{A} = \mu \frac{dy}{dy}, \quad F = \frac{\mu A}{t}\dot{x} \quad \boxed{\frac{\mu A}{t} = c}$

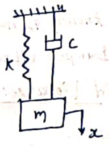
$\boxed{F = c\dot{x}} \approx \boxed{F \propto \dot{x}}$

First Question is Compulsory and out of remaining

Analysis for Damped System:-

By using Dalembert Principle





$m\ddot{x} + K\dot{x} + Kx = 0$

$\ddot{x} + \frac{K\dot{x}}{m} + \frac{k}{m}x = 0$

$\boxed{\ddot{x} + \frac{c\dot{x}}{m} + \omega_n^2 x = 0}$ —① fig:-Free body Diagram

Equation of motion of free damped system.

$\boxed{\ddot{x} + \frac{c}{m}\cdot\dot{x} + \omega_n^2 x = 0}$ —②

This is IInd order Linear differential equation.

Solution of this equation CF + (PI) X

$x = A\cdot e^{\alpha_1 t} + Be^{\alpha_2 t} \quad (\alpha_1 \neq \alpha_2)$

or

$x = (A+Bt)e^{\alpha t} \quad (\alpha_1 = \alpha_2 = \alpha)$

{ A & B are constant which is found by initial cond^n. }

$\alpha_{12} \rightarrow$ Root of auxilary equation $(\alpha_1 \& \alpha_2)$

If we assume $x = e^{\alpha t}$ then

$\dot{x} = \alpha e^{\alpha t}$ & $\ddot{x} = \alpha^2 e^{\alpha t}$ then from equation ②

$\boxed{\alpha^2 + \frac{c}{m}\alpha + \omega_n^2 = 0}$ —③

↳ Quardic equation

Now $\alpha_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\alpha_{12} : \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\times1\times\omega_n^2}}{2\times1}$

$\alpha_{12} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega_n^2}$

(Zeta) $\xi = \sqrt{\frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2}}$ — Epsilon

$\xi = \sqrt{\frac{c^2}{4m^2} \Big/ \frac{k^2}{mk}}$

$\xi = \sqrt{\frac{c^2}{4m^2} \times \frac{M}{K}}$

$\boxed{\text{(Zeta)}\ \xi = \frac{c}{2\sqrt{Km}}}$

$\boxed{2\xi\omega_n = \frac{c}{m}}$

finally the equation of motion

$\boxed{\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 = 0}$

$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \quad \alpha_1 \neq \alpha_2$

or

$x = (A+Bt)e^{\alpha t} \quad$ if $\alpha_1 = \alpha_2 = \alpha$.

Now

$\alpha_{12} = -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - \omega_n^2}$

$\alpha_{12} : \omega_n\left(-\xi \pm \sqrt{\xi^2 - 1}\right)$

$\boxed{x = Ae^{(-\xi + \sqrt{\xi^2-1})\omega_n t} + Be^{(-\xi - \sqrt{\xi^2-1})\omega_n t}}$

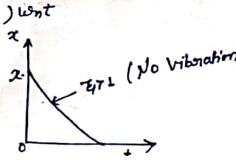
- If $\xi > 1$ (System is over damped system) ↓ Coulomb damping.
- If $\xi = 1$ (Critical damped system)
- If $\xi < 1$ (Under damped system / viscous damping)

1 Over damped system $(\xi > 1)$:-

$\alpha_1 \neq \alpha_2$

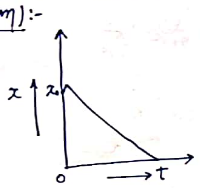
$x = Ae^{(-\xi + \sqrt{\xi^2-1})\omega t} + Be^{(-\xi - \sqrt{\xi^2-1})\omega_n t}$

there A & B constant $e^{-()}$ so graph is super negative decaying function.



2 When $\xi = 1$ (Critical damped system):-

$$\boxed{\alpha_1 = \alpha_2}$$

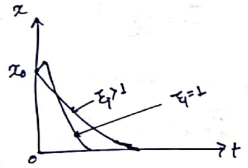


$x = (A+Bt)e^{-\alpha t}$

$x = (A+Bt)e^{-\xi \omega n t}$

Note:- ~~Critically~~ (i) The response of Critically damped is very fast than the over damped system.

(ii) The concept of critically damped System is use in AK-47



The concept of over damped system is used in Door Closer closers.

3 Under damped System ($\xi < 1$):-

$\alpha_1 \neq \alpha_2$

$x = A e^{(-\xi + \sqrt{\xi^2 - 1})\omega n t} + B e^{(-\xi - \sqrt{\xi^2-1})\omega n t}$

$x = A e^{+\alpha_1 t} + B e^{\alpha_2 t}$ ↗

$x = A e^{-\xi \omega n t + (\sqrt{\xi^2-1})\omega n t} + B e^{-\xi \omega n t - (\sqrt{\xi^2 - 1})\omega n t}$

$= e^{-\xi \omega n t} \left[A e^{(\sqrt{\xi^2-1})\omega n t} + B e^{(-\sqrt{\xi^2-1})\omega n t} \right]$

$= e^{-\xi \omega n t} \left[A e^{(i\sqrt{1-\xi^2})\omega n t} + B e^{(-i\sqrt{1-\xi^2})\omega n t} \right.$

$. \ e^{-\xi \omega n t} [A (\cos(\sqrt{1-\xi^2})\omega n t + i \sin(\sqrt{1-\xi^2})\omega n t) +$

$B(\cos(\sqrt{1-\xi^2})\omega n t - i \sin(\sqrt{1-\xi^2})\omega n t)]$

* Critical Damping system (C_c):-

when $\xi = 1$ then

$\cancel{2} \times \xi \times \omega n = \frac{c}{\cancel{m}}$

$\cancel{2} \times 1 \times \omega n = \frac{C_c}{\cancel{m}}$

$$\boxed{\xi = \frac{c}{C_c}} = \frac{\text{Actual damping constant}}{\text{Critical damping constant}}$$

Num:- A vibatory system in define the fallowing factor

$m = 3kg, \ K = 100 \ N/m . \ c = 3 \frac{N-S}{m}$

Determine

(i) The damping factor

(ii) Natural freequency of damped vibration.

(iii) Logarithm decrement.

(iv) Ratio of two consecutive Amplitude.

(v) The Number of cycle after which the original amplitude is reduce to 20%.

Solution:-

(i) Damping factor

$\xi = \frac{c}{2 \omega n \times m} = \frac{c}{2 \times m} \times \sqrt{\frac{m}{K}} = \frac{3}{2 \times 3} \times \sqrt{\frac{3}{100}}$

~~$\xi = 8.66$~~

$2 \times 5.77 \times \xi = \frac{3}{3}$

$$\boxed{\xi = 0.086}$$

(ii) $\omega d = \omega n \sqrt{1 - \xi^2}$

$\omega d = 5.75$ ~~Hz~~ rad/sec $\quad T = \frac{2\pi}{5.75} = 0.99Hz$

(iii) logarithmic decrement:-

$$\delta = \ln\frac{x_0}{x_1} = \frac{2\pi\xi_1}{\sqrt{1-\xi_1^2}} = \frac{1}{n}\ln\frac{x_n}{x_{n+1}}$$

$$\delta = \frac{2\times3.14\times0.086}{\sqrt{1-0.086^2}} = \frac{0.54}{0.99} = 0.54$$

(iv) $\frac{x_1}{x_2} = e^{0.54} = 1.71$ from $\delta = \ln\frac{x_0}{x_1}, \frac{x_1}{x_2}$

(v) $\delta = \frac{1}{n}\ln\frac{x_1}{x_1/5}$

$$n = \frac{1}{\delta}\ln\frac{5\times x_1}{x_1}$$

$$\boxed{n = \frac{1}{0.54}\ln 5 = 2.98}$$

Q:- Torsional pendulum $I = 600$ kg-cm2 immersed in viscous fluid Brass shaft. $d = 10cm$, $l = 40$ cm long. When pendulum is vibrating the observed amplitude on same side of rest position for successive cycles are 9°, 6° & 4°.

Determine (a) $\delta = ?$

(b) Damping torque per unit velocity.

(c) Periodic time of vibration

where: $G = 4.5\times10^{10}$ N/m²

Solution:- $\delta = \ln\frac{\theta_1}{\theta_2}$

$$\delta = \ln\frac{9}{6} = .40$$

(b) Damping torque per unit velocity:-

T=c

$$2\xi_1\omega_n = \frac{c}{m}$$

$$2\xi_1\sqrt{\frac{K_t}{I}} = \frac{c}{I}$$

$$K_t = \frac{GJ}{l} = \frac{4.5\times10^{10}\times\pi\times(0.1)^4}{0.4\times32}$$

$$\boxed{K_t = 0.42}$$

$$\delta = \frac{2\pi\xi_1}{\sqrt{1-\xi_1^2}} = \frac{2\times3.14\times\xi_1}{\sqrt{1-\xi_1^2}} = .40$$

$$\frac{39.43\,\xi_1^2}{1-\xi_1^2} = .40$$

$$\xi_1^2 = 0.010 - 0.01\xi_1^2$$

$$1.01\xi_1^2 = 0.01$$

$$\boxed{\xi_1^2 = 0.099}$$

$\delta = 0.40$

$$\delta = \frac{2\pi\xi_1}{\sqrt{1-\xi_1^2}} : \quad 0.40 = \frac{2\times3.14\times\xi_1}{\sqrt{1-\xi_1^2}} = \frac{39.43\xi_1^2}{1-\xi_1^2} = 0.16$$

$$\Rightarrow 0.16 - 0.16\xi^2 = 39.43\xi_1^2$$

$$\boxed{\xi^2 = 0.063}$$

$$k_t = \frac{4.5\times10^{10}\times3.14\times(.1)^4}{0.4\times32} = 110390\,6.25$$

$$\omega_n = \sqrt{\frac{1103906.25}{600}}\times10^4$$

$$\boxed{c = 32.42\ \frac{N\text{-}m}{rad}}$$

(c) Periodic time of vibration

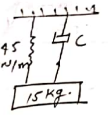
$$T_d = \frac{2\pi}{\omega_d}, \quad \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 4289.33\sqrt{1-(0.063)^2}$$

$$\boxed{T_d = 1.46\times10^{-3}sec}$$

Num: for the system shown in fig. the chrecutrstics of the ~~destpat~~ dashpat is such that when a constant force of ~~90~~ 49 N is applied to the ~~f~~ piston its velocity is found to be constant at 0.12 m/sec determine the ~~i~~ Value of c

(i) Would you aspect the complete system to be periodic or Aperiodic.



Solution:-

$$C = \frac{F}{\dot{x}} = \frac{49}{0.12} = 408.33 \frac{N\text{-}S}{m}$$

$$\text{given } \xi = \frac{c}{m}$$

$$\xi \times 2 \times \sqrt{\frac{245 \times 10^3}{15}} = \frac{408.33}{15}$$

$$\xi = \frac{408.33}{15 \times 8.08}$$

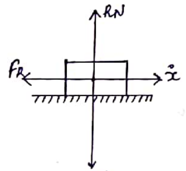
$$\xi = 3.361$$

$$\boxed{\xi > 1 \text{ System is aperiodic}}$$

* Coulamb Damping:-

When one body is allow to slide over the other a surface of one body offer some resistance to the movement of the other body on it this resistance force is called force of friction.

Some amount of energy is westage in over coming this friction thus the surface are dry so the amount of coulomb damping is.



The constant of columb damping:

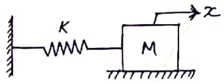
$$\boxed{fr \propto RN} \; ; \; \boxed{fr = \mu RN}$$

μ = Co-efficient friction.

fr acts opposite to the direction of motion.

The three posible condition are:-

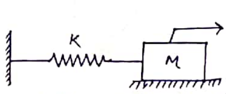
(i)



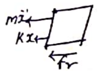
$$\boxed{m\ddot{x} + kx = 0}$$

Equilibrium Position.

(ii)

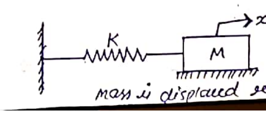


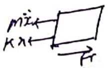
$$\boxed{m\ddot{x} + kx + fr = 0}$$



mass is displaced to the right & moving to the right

(iii)





$$\boxed{m\ddot{x} + kx - fr = 0}$$

mass is displaced ~~right~~ & moving to the left.

Let equation (iii)

$$m\ddot{x} + kx - fr = 0$$

$$\ddot{x} + \frac{k}{m}\left(x - \frac{fr}{k}\right) = 0$$

let

$$\begin{cases} x - \frac{fr}{k} = y, \\ \ddot{x} = \ddot{y} \end{cases}$$

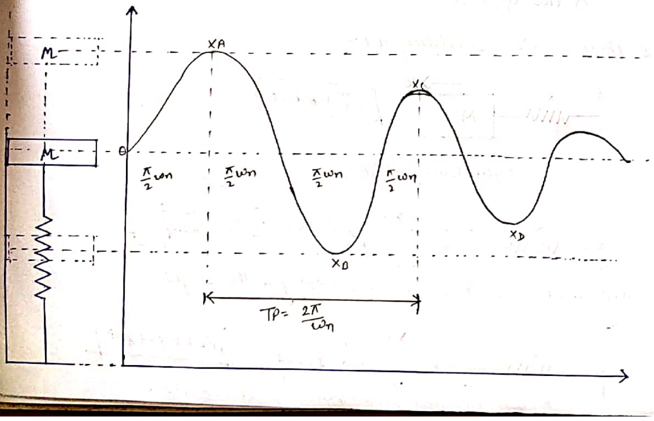
$$\boxed{\ddot{y} + \frac{k}{m}y = 0}$$ —(iv) { Equation of motion of ~~cot~~ coulomb damping}

from Eqn. (iv) $\boxed{\omega_n = \sqrt{\frac{k}{m}}}$

$$x - \frac{fr}{k} = y$$ where assume $y = 0$

$$\boxed{x = \frac{fr}{k}}$$

$\boxed{\ddot{y} + \frac{k}{m}y = 0}$ → This equation is hold is good for quarter of the Cycle $\boxed{x = \frac{fr}{k}}$



$$(P.E)_A = \frac{1}{2} k x_A^2$$

$$(P.E)_B = \frac{1}{2} k x_B^2$$

$$(\Delta)\ \text{Loss} = \frac{1}{2} k (x_A^2 - x_B^2)$$

$$\text{workdone} = fr \times (x_A + x_B)$$

$$\frac{1}{2} k (x_A^2 - x_B^2) = fr \times (x_A + x_B)$$

$$\frac{1}{2} k (x_A - x_B) = fr$$

$$\boxed{x_A - x_B = \frac{2fr}{k}}$$

$$x_A - x_C = (x_A - x_B) + (x_B - x_C)$$

$$\frac{2fr}{k} + \frac{2fr}{k}$$

$$\boxed{x_A - x_C = \frac{4fr}{k}}$$

* Hysteresis Damping (Structural Damping):-

It is the inherent chrecteristic of the material and the resistance is offer by the elastic propertys from within the body there is intermolecular friction in there structure which oppeses its movement the magnitude of this damping is very small as campair to other damping. expriment show that for elastic material for loading & unloading condition a loop is form on stress strain curve this loop is called hysteresis loop.

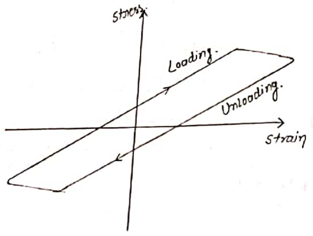
The area of loop is the amount of energy dissipated in one cycle during vibration. The energy loss per cycle is expressed as

$$\boxed{E = \pi k \alpha A^2}$$

A → amplitude of vibration

α → dimensionless damping factor

Engineering Thermodynamics
First Question is Compulsory and out of remaining
attempt any four question.



UNIT = III

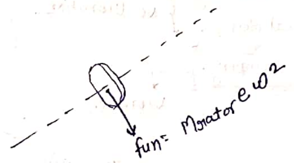
farced damped Vibration:-

- mass
- stiffnen
- kinematic friction → farced damped vibration
- $f_{un} \neq 0$ (Unbalance force).

Ratating / Reciprocating.

*Vibration causing unbalance in Runing machine:-

Ratating unbalance:-



$f_{un} = M_{rotar} e\omega^2 \times \sin\omega t$

$\boxed{f_{un} = f_o \sin\omega t}$

f_o → maximum value

$\omega = \frac{2\pi N}{60}$ → speed of rotor

ω → force frequency & Excitation frequency

Reciprocating unbalance

$\boxed{\text{Crank Radius} = \frac{\text{Stroke}}{2}}$

$\boxed{f_{un} = M_{reciprocating} \times r \times \omega^2}$

*

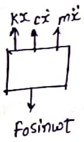
attempt any four question.

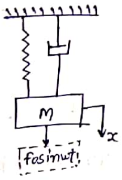
Forced damped Vibration:- (Running machine analysis):-

$$\boxed{m\ddot{x}+c\dot{x}+kx=f_0\sin\omega t}$$

Inertia force

Damping force





$$\ddot{x}+\frac{c}{m}\dot{x}+\frac{kx}{m}=f_0\sin\omega t$$

This is the second order linear differential equation

$x = CF+PI$

$CF = \ddot{x}+\frac{c}{m}\dot{x}+\frac{k}{m}x=0$

$\hookrightarrow = \ddot{x}+2\xi\omega_n\dot{x}+\omega_n^2 x=0$

CF
- $\xi>1$ over damping } No vibration.
- $\xi=1$ critical damping
- $\boxed{\xi<1.\ \text{under damping.}}$ → vibration

} Decay with time

$$PI=\frac{\frac{f_0}{m}\sin\omega t}{D^2+(2\xi\omega_n)D+\omega_n^2}$$

D- Differential

$$PI=\frac{f_0/m\ \sin\omega t}{-\omega_n^2+(2\xi\omega_n)D+\omega_n^2}\times$$

$$PI=\frac{f_0/m\ \sin\omega t}{(\omega_n^2-\omega^2)+(2\xi\omega_n)D}\times\frac{(\omega_n^2-\omega^2)-(2\xi\omega_n)D}{(\omega_n^2-\omega^2)-(2\xi\omega_n)D}$$

$$PI=\frac{\frac{f_0}{m}\left[(\omega_n^2-\omega^2)\sin\omega t-(2\xi\omega_n\omega)\cos\omega t\right]}{(\omega_n^2-\omega^2)^2+(2\xi\omega_n\omega)^2}$$

$\omega_n^2-\omega^2 = A\cos\phi$

$2\xi\omega_n\omega = A\sin\phi$

$$PI=\frac{f_0/m\left[\sin(\omega t-\phi)\right]A}{A^2}$$

$$=\frac{f_0/m\ \sin(\omega t+\phi)}{\sqrt{(\omega_n^2-\omega^2)^2+(2\xi\omega_n\omega)^2}}$$

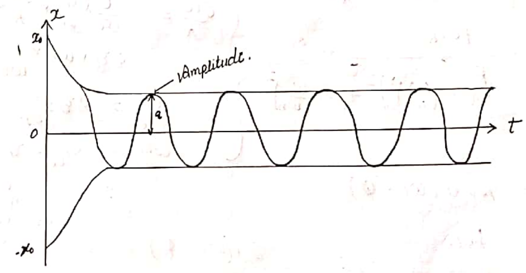
$$=\frac{\frac{f_0}{m}\sin(\omega t+\phi)}{\sqrt{\omega_n^4\left[\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2\right]}}$$

$$=\frac{f_0/m\ \sin(\omega t+\phi)}{\frac{k}{m}\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$\omega_n = K/m$

$$\boxed{PI=\frac{f_0/k\ \sin(\omega t+\phi)}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}}$$

→ Does not depend on time. (Independent on time).



Assume-

$x_p = A\sin(\omega t-\phi)$

$\dot{x}_p =$ ~~$\omega A(\cos$~~ $\omega A\cos(\omega t-\phi)$

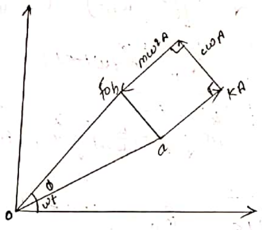
$\dot{x}_p = \omega A\sin\left(\pi/2-(\omega t-\phi)\right)$

$\ddot{x}_p = \omega^2 A\sin\left(\pi-(\omega t-\phi)\right)$

$m\omega^2 A\sin(\pi-(\omega t-\phi)) + c\omega A\sin(\pi/2-(\omega t-\phi)) + kA\sin(\omega t-\phi)$

$-f_0\sin\omega t = 0$

from fig:-



$f_0^2 = (kA-m\omega^2A)^2 + (c\omega A)^2$

$$\boxed{A^2 = \frac{F_0^2}{(k-m\omega^2)^2+(c\omega)^2}}$$

$$A = \frac{f_0 1}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}}$$

$$A = \frac{f_0/k}{\sqrt{(\cancel{k}-\frac{m\omega^2}{k})^2+(\frac{c\omega}{k})^2}}$$

$$A = \frac{f_0/k}{\sqrt{\left(1-\left[\frac{\omega}{\omega_n}\right]^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$\frac{c\omega}{k} = \frac{c}{c_c}\times\frac{c_c}{2m}\times\frac{2m}{k}\times\omega$

$2\xi\times\omega_n\times\frac{1}{\omega_n^2}\times\omega$

$\frac{c\omega}{k} = 2\xi\frac{\omega}{\omega_n}$

where $c/c_c = \xi$

$\because 2\times\omega_n = \frac{c_c}{m}$

$\omega_n = \frac{c_c}{2m}$

$x_p = P.I = A\sin(\omega t-\phi)$

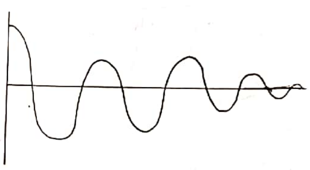
$$P.I = \frac{f_0/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}\sin(\omega t-\phi)$$

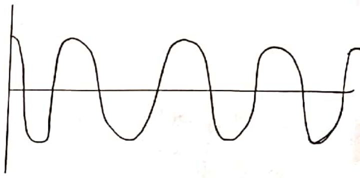
Total Response:-

Final solution of the system is

$x = x_c + x_p$
$\quad\;\downarrow\quad\;\downarrow$
$\quad C.I\quad P.I$

The first part of this equation vanishs with time while the second part remain into existance the amplitude remain constant due to second part and it is cared steady vibration and the first part is known as ~~transit~~ transient vibration.





* Vibration in running m/c will never stop.
* To improve the effeciency of m/c fatique life or running life is the important factor.

$$\text{Magnification factor} = \frac{A}{F_0/k} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\boxed{A\rightarrow\frac{\omega}{\omega_n}\cdot\frac{\omega}{\omega_n}=1\text{ (Resonance)}}$$

Frequency Response curve & phase frequency Response center:-

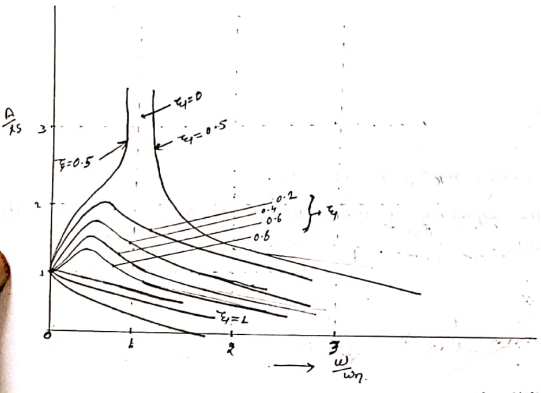
attempt any four question.

frequancy response curve:-

(A curve b/w frequmncy ratio $\frac{\omega}{\omega_n}$ magnification factor (A/As is known as frequency response curve:

The curve drawn with the help fallowing equation

$$\frac{A}{(f_0/k)} = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2-\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$



- The fallowing points are observed from the curve

1→ At zero frequency ratio magnification factor is unity & no effect of damping on system

2→ Damping reduces magnification factor for each value of frequency

3→ Maximum value of amplitude accurs at left of resonance,

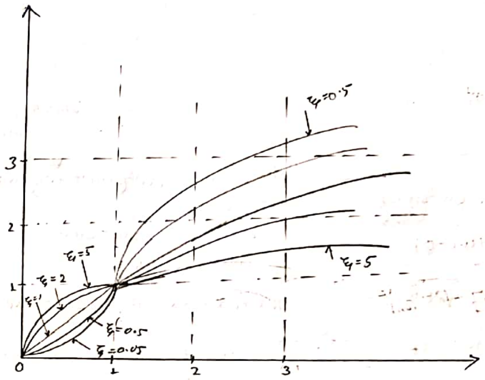
4→ Magnification factor is below unity for all value of damping greater then 1

* phase frequency ~~curve~~ Response curve:-

The curve b/w phase angle (ϕ) and frequency ratio $\frac{\omega}{\omega_n}$ is known as phase frequency response curve.

The curve plotted with the help of equation fallowing equation

$$\boxed{\tan\phi = \frac{2\xi_1\,\omega/\omega_n}{1-\left(\frac{\omega}{\omega_n}\right)^2}}$$



- Fallowing point observed from above curve:-

1. At $\omega/\omega_n=1$ (resonent frequency) at resonance frequency phase angle is

2. Phase angle Increases far decreasing value of damping beyond resonance.

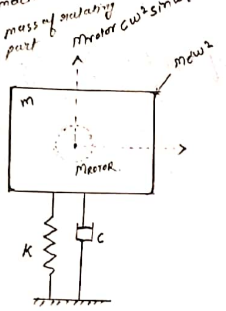
3. Increasing in phase angle (ϕ) far increasing damping below resonance.

4. The system is undamped if ϕ either zero degree or 180°

Response of Rotating and reciprocating unbalance:-

m→ Mass of machine.

Mrotor - mass of rotating part



Fc= Mrotor e ω² sinωt

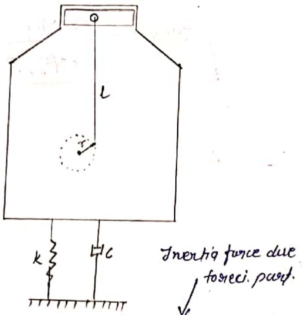
Equation of motion.

$m\ddot{x}+c\dot{x}+kx = M_{rotor} e\omega^2 \sin\omega t$.

$$P.I = \frac{\overset{M_{re}\omega^2\sin\omega t}{F_0/k} \sin(\omega t-\phi)}{\sqrt{[1-(\omega/\omega_n)^2]^2+(2\xi\frac{\omega}{\omega_n})^2}}$$

m→ mass of machine

Mreci→ mass of reciprocating part of machine.



$F_c = M_{reci}\, r\omega^2(\sin\omega t + r/_l \sin 2\omega t)$

$l >> r$

$F_c = M_{reci} \cdot r\omega^2 \sin\omega t$.

Equation of motion.

$m\ddot{x}+c\dot{x}+kx = M_{reci}\, r\omega^2 \sin\omega t$

$$P.I = \frac{M_{reci}\, r\omega^2 \sin\omega t \; \sin(\omega t-\phi)}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2+(2\xi\frac{\omega}{\omega_n})^2}}$$

$\boxed{\text{When } l = r}$

$$A = \frac{\frac{m_0 e\omega^2}{k}\times\frac{m}{m}}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2-(2\xi\frac{\omega}{\omega_n})^2}} = \frac{\frac{m_0 e}{m}\times\frac{m}{K}\times\omega^2}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2-(2\xi\frac{\omega}{\omega_n})^2}}$$

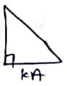
$$\frac{A}{\frac{m_0 e}{m}} = \frac{(\omega/\omega_n)^2}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2-(2\xi\,\omega/\omega_n)^2}}$$

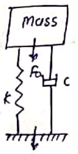
$$\boxed{\phi = \tan^{-1}\frac{2\xi\,\omega/\omega_n}{1-(\frac{\omega}{\omega_n})^2}}$$

Vibration isolation & Force transmissiblity :-

(Safety of Surrounding)

As we know that the maximum Spring force and damping force are mutually perpendicular to each other So the resultant of these forces is to transmitted in the foundation.





F_T= maxi force transmid the ground,

$$f_T = \sqrt{(kA)^2+(c\omega A)^2} \text{ from } m\ddot{x}+c\dot{x}+mx-F_0\sin\omega t=0$$

$$\begin{cases} x = A\sin\omega t \\ \dot{x} = \\ \ddot{x} = \end{cases}$$

$$f_T = kA\sqrt{1+\frac{(\frac{c\omega}{m})^2}{(k/m)^2}}$$

$$F_T = kA\sqrt{1+(2\xi\frac{\omega}{\omega_n})^2}$$

~~fo=~~ ~~AK~~ Now the maximum unbalance force produce by the ~~the~~ rotating component of m/c. which is given by.

$$\boxed{f_0 = AK\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2+(2\xi\frac{\omega}{\omega_n})^2}}$$ from previous Article

$\boxed{\varepsilon = \frac{f_T}{f_0}}$ → Ratio of f_T/F_0 is know as force transmissiblity

~~The Ratio f.~~

$$\varepsilon = \frac{\sqrt{1+(2\xi\,\omega/\omega_n)^2}}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2+(2\xi\frac{\omega}{\omega_n})^2}}$$

We learnt:-

$\boxed{0 \leq \varepsilon = \frac{f_T}{F_0} < 1}$

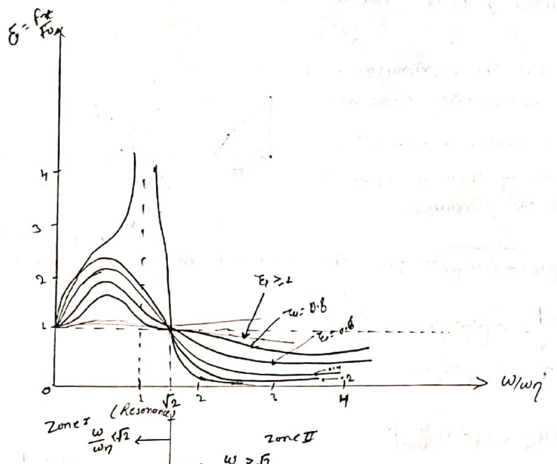
$\varepsilon = \omega_n/\omega$

$\varepsilon_0 =$ ~~ξ~~ ξ

for all of ξ

$\frac{\omega}{\omega_n} = 0, \varepsilon = 1$

$\frac{\omega}{\omega_n} = \sqrt{2}, \varepsilon = 1$



Conclusion:-

1. Under ~~damped~~ damping:- ↑ ε_1 ↓

$\boxed{\varepsilon = \frac{F_T}{F_0}}$

$\frac{\omega}{\omega_n} < \sqrt{2} = \varepsilon > 1 \; (\varepsilon \uparrow)$

Best $\varepsilon = 0$
ε is very low

$\frac{\omega}{\omega_n} > \sqrt{2} = \varepsilon_1 < 1 \; (\varepsilon \downarrow)$

$\frac{\omega}{\omega_n} = \sqrt{2} \Rightarrow \varepsilon = 1 \; (\varepsilon = \text{constant})$

2. Vibration isolation will be effactive :- $(\varepsilon_1 < 1)$

$\boxed{\frac{\omega}{\omega_n} > \sqrt{2}}$

Nate :- But mostly machines are work in $\boxed{\frac{\omega}{\omega_n} < \sqrt{2} \text{ Zone}}$

3. In effective vibration isolation zone

$\left(\frac{\omega}{\omega_n} > \sqrt{2}\right)$

- No damping is best, • Damping is determental (harmful)

Conclusion:-

working zone	Need of external Isolation	How to make it.
• If $\frac{\omega}{\omega_n} > \sqrt{2}$	less	Less damping material is Required.
• If $\frac{\omega}{\omega_n} < \sqrt{2}$	more	high damping material
• If $\frac{\omega}{\omega_n} = \sqrt{2}$.	medium damping is required	

⇒ Vibration isolation:-

The high speed engines and machine when mountaned on foundation & suppert causes vibration of excessive amplitude becau of unbalancing. forces setup during there working. these forces ~~damg~~ demage the foundation on which the m/c are mountined so the vibration transmitted to the foundation. should be reduse considrebly by using some devices such type spring demper etc.

Material for Isolation:-

1 Rubber → • quiet useful far shear loading.
- Sound transmitiblity is very low.
- Can't be use at high temp.
- Prefere far light loads & high frequency & oscillation.
- ~~falt felt~~

2 felt:- • Damping factor of falt is high so it is use far low frequency ratio.
- It is use in the form of pads.

3. Cork :- Suitable far compressive load.
- It not ~~effactly~~ perfactely elastic.
- At high load it becomes more flexible.

4 Spring .

4 Spring:- metal Spring used are of two type helical & leaf Spring.

- High Sound transminiblity.
- Usefull for high frequency ratio.
- Can be used in all working condition.

motion Transminiblity:-

In case of locomotive or vehicles the wheel at as base ~~suppose~~ or support for the system the wheel can move vertically up and down on the road surface during the motion of the vehicle at the same time their is relative motion between the wheel and chesis so chesis having motion relative to the wheel & wheel core having motion relative to road surface the amplitude of vibration in case of support motion depends on the speed of vehicle & nature of road surface.

forced Vibration due to excitation of support:-

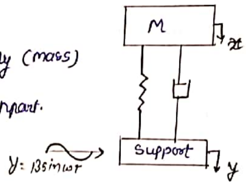
In a vibratory system weight the support is put to excitation absolute or relative motion became important.

Absolute motion:- Absolute motion of a mass means its motion with respect to the co-ordinate system attached to the earth

$y = B\sin\omega t$ = absolute displacement of Support.

$x \rightarrow$ Absolute displacement of body (mass)

$z \rightarrow$ Relative motion of mass & support.



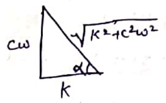
$$m\ddot{x} + c(\dot{x} - \dot{y}) + K(x - y) = 0$$

$$m\ddot{x} + c\dot{x} + Kx = Ky + c\dot{y}$$

Assume. $y = B\sin\omega t$

$\dot{y} = B\omega\cos\omega t$

$$m\ddot{x} + c\dot{x} + kx = KB\sin\omega t + cB\omega\cos\omega t$$



$$m\ddot{x} + c\dot{x} + Kx = B(K\sin\omega t + c\omega\cos\omega t)$$

$$m\ddot{x} + c\dot{x} + Kx = \sqrt{K^2 + c^2\omega^2}\left[\frac{K\sin\omega t}{\sqrt{K^2 + c^2\omega^2}} + \frac{c\omega\cos\omega t}{\sqrt{K^2 + c^2\omega^2}}\right]$$

$$m\ddot{x} + c\dot{x} + Kx = \sqrt{K^2 + \omega^2 c^2}\,[\sin\omega t\cos\alpha + \cos\omega t\sin\alpha]$$

$$m\ddot{x} + c\dot{x} + Kx = \sqrt{K^2 + c^2\omega^2}\,[\sin(\omega t + \alpha)$$

$$\tan\alpha = \frac{c\omega}{K}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{c\omega}{K}\right)}$$

Comparing the above equation from this equation:-

$$PI = \frac{F_0/K\ \sin(\omega t - \phi)}{\sqrt{[1-(\omega/\omega_n)^2]^2 + (2\xi\,\omega/\omega_n)^2}}$$

$$A = \frac{B\sqrt{K^2 + c^2\omega^2}/K}{\sqrt{[1-(\omega/\omega_n)^2]^2 + (2\xi\,\omega/\omega_n)^2}}$$

$$A/B = \frac{\sqrt{1 + (2\xi\,\omega/\omega_n)^2}}{\sqrt{[1-(\omega/\omega_n)^2]^2 + (2\xi\,\omega/\omega_n)^2}}$$

$\boxed{\text{Graph: same}}$

The ratio A/B is called the displacement transmissiblity which is the ratio of the amplitude of the body to the amplitude of the support

Relative motion:- Let z is the relative displacement of mass with respect to support

$z = (x-y)$, $\dot{z} = (\dot{x}-\dot{y})$, $\ddot{z} = (\ddot{x}-\ddot{y})$

$m\ddot{x} + c(\dot{x}-\dot{y}) + k(x-y) = 0$

$m(\ddot{z}+\ddot{y}) + c\dot{z} + kz = 0$ $\quad \because y = B\sin\omega t$

$\ddot{y} = -B\omega^2 \sin\omega t.$

$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$

$m\ddot{z} + c\dot{z} + kz = mB\omega^2 \sin\omega t.$

This equation known as relative equation of motion & it is the same form.

$m\ddot{x} + c\dot{x} + kx = me\omega^2 \sin\omega t$

$$Z = \frac{mB\omega^2/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

where $Z \rightarrow$ Relative amplitude of mass support.

$$\frac{Z}{B} = \frac{m\omega^2/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

Q:1

$m = 100\text{kg}.$

$k = 7.5 \times 10^5 \text{N/m}.$

$F_0 = 392 \text{ N}$

at speed 3000 rpm.

$\xi = 0.2$ (i) Determine the amplitude of motion due to unbalance.

(ii) Transmissibility

(iii) Transmitted force.

$$A = \frac{F_0/k}{\sqrt{\left[1-\left(\omega/\omega_n\right)^2\right]^2 + \left(2\xi\,\omega/\omega_n\right)^2}}$$

$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 3000}{60} = 314$

$\omega_n = \sqrt{k/m} = \sqrt{\frac{7.5 \times 10^5}{100}} = 86.60$

$$A = \frac{392/7.5\times10^5}{\sqrt{\left[1-\left(\frac{314}{86.60}\right)^2\right]^2 + \left(2\times0.2\times\frac{314}{86.60}\right)^2}}$$

$= ~~147.54 / 2.10~~$

$= \frac{392}{}$ $\quad \boxed{A = \cancel{4.2\times10^2} \; 0.0497}$ $\quad 4.14\times10^{-5}$

Trans.

$\frac{F_T}{F_0} = \varepsilon$

$$\frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} = \frac{\sqrt{1+\left(2\times0.2\times\frac{314}{86.10}\right)^2}}{\sqrt{\left[1-\left(\frac{314}{81.10}\right)^2\right]^2 + \left(2\times0.2\times\frac{314}{86.60}\right)^2}}$$

$\boxed{\varepsilon = 0.15}$

Transmitted force $\Rightarrow$ $F_T = \varepsilon \times F_0 = 78.33$

Q $m = 70\,kg.$

$k = 14 \times 10^5\ N/m$

$\xi = 0.20$

$m_0 = 2\,kg.$

Stroke length: 0.08m

Speed N = 2700 rpm

(crank Radius) = $\frac{\text{Stroke length}}{2}$

$\omega_n = \sqrt{\frac{k}{m}} = 141.42$

(i) Amplitude

(ii) Vibratory force.

$$A = \frac{m_0 e\omega^2 / k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$= \frac{2 \times 0.04 \times (282.6)^2}{14\times10^5 \sqrt{\left[1-\left(\frac{282.6}{141.42}\right)^2\right]^2 + \left[2\times0.2\times\frac{282.6}{141.42}\right)^2}}$$

$$= \frac{6389.02}{14\times10^5 \times \sqrt{8.95 + 0.63}}$$

$$= \frac{6389.02}{43.35\times10^5}$$

$$\boxed{A = 1.47\times10^{-3}\,m}$$

$$f_T = \sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2} = kA\sqrt{1 + 2\times0.2\times\left(\frac{282.6}{141.42}\right)^2}$$

$$= 2634.$$

Q $k = 6400\,N/m$

F = 500 N at 4.0 m/sec

$m_0 = 0.5\,kg.$

$r = 5\,cm$

$m = 20\,kg.$

N = 4000 rpm.

Determine → (i) Damping factor

(ii) Amplitude of vibration

(iii) Phase angle.

(iv) Resonant Speed & Resonant amplitude.

(v) forces excerted by the Springs dashpot on the motor

Solution:-

$$\xi = \frac{c}{m} = 2\xi\omega_n$$

$$\xi = \frac{c}{2m\omega_n}$$

$f = c\dot{x} =$

$$c = F/\dot{x} = \frac{500}{4} = 125$$

$$\xi \cancel{= \frac{125}{2\times20\times418.66} = 7.4\times10^{-3}} \qquad \xi = \frac{c}{2\sqrt{km}}$$

$$\boxed{\xi = 0.174}$$

(ii)

$$A = \frac{m_0 r\omega^2}{k\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} = \frac{0.5\times0.05\times(418.66)^2}{6400\sqrt{\left[1-\left[\frac{418.66}{17.88}\right]\right]^2 + \left(2\times0.174\times\frac{418.81}{17.88}\right)^2}}$$

23.41

$$= \frac{4381.90}{\sqrt{299495.36 + 66.10}} \approx 3502864.29$$

$$\boxed{A = 1.18\times10^{-3}}$$

$$\phi = \tan^{-1}\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \tan^{-1}\frac{2\times.174\times\frac{418.11}{17.88}}{1-\left(\frac{418.66}{17.88}\right)^2}$$

$$\phi = 19.79$$

attempt any four question.
1. Write Short note on

~~UNIT IV~~

~~Never maximizing method~~:-

Resonance amplitude & Speed $\omega/\omega_n = 1$

Speed $\omega = \omega_n$. & $\boxed{\frac{A}{\left(\frac{m_0 r}{m}\right)} = \frac{1}{2\xi_1}}$

* The frequency corresponding to the peak amplitude take $c = 40$

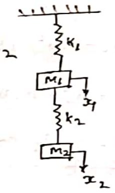
$$A/x_s = \frac{1}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2 + (2\xi_1\frac{\omega}{\omega_n})^2}}$$

$$\frac{d(A/x_s)}{d(\omega/\omega_n)} = \boxed{\frac{\omega_{max}}{\omega_n} = \sqrt{1-2\xi_1^2}}$$

$$\boxed{\omega_{max} = \omega_n\sqrt{1-2\xi_1^2}}$$

UNIT- IV th

Two DOF System:-



Determine the two natural frequency of vibration & the ratio of amplitudes of the masses m_1 & m_2 for the two modes of vibration.

$m_1 = 1.5\,kg$, $m_2 = 0.8\,kg$.

$k_1 = k_2 = 40\,N/m$.

Solution:-

Equation of motion

for mass m_1

~~$m_1\ddot{x}_1 + k_1x$~~ $m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = 0$ —— ①

for mass m_2

$m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0$ —— ②

Rearranging the equation ① & ②.

$(m_1\ddot{x}_1 + x_1(k_1 + k_2) - k_2x_2 = 0$ —— ③

$m_2\ddot{x}_2 - k_2x_1 + k_2x_2 = 0$ —— ④

$x_1 = A_1\sin\omega t$. $\ddot{x}_1 = -A_1\omega^2\sin\omega t$

$x_2 = A_2\sin\omega t$ $\ddot{x}_2 = -A_2\omega^2\sin\omega t$.

from equation ③

$$\boxed{(k_1 + k_2 - m\omega^2)A_1 - k_2A_2 = 0}$$ —— ⑤

from equation 4

~~$k_2A_2 - (k_2 + m_2\omega^2)A_1 = 0$~~

for equation 4

$-k_2A_1 + (k_2 - m_2\omega^2)A_2 = 0$ —— ⑥

$$\left(\frac{A_1}{A_2}\right)_1 = \frac{k_2}{(k_1 + k_2 - m_1\omega^2)} =$$

$$\left(\frac{A_1}{A_2}\right)_2 = \frac{k_2 - m_2\omega^2}{k_2}$$

$$\begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} \begin{bmatrix} (k_1+k_2-m_1\omega^2) & -k_2 \\ -k_2 & (k_2-m_2\omega^2) \end{bmatrix} = 0$$

$(k_1+k_2-m_1\omega^2)\,(k_2-m_2\omega^2) - k_2^2$

$$\boxed{\begin{gathered} k_1k_2 + \cancel{k_2^2} - k_2m_1\omega^2 - k_1m_2\omega^2 - k_2m_2\omega^2 + m_1m_2\omega^4 - \cancel{k_2^2} = 0 \\ k_1k_2 - \omega^2(k_2m_1 + k_2m_2) - k_1m_2\omega^2 + m_1m_2\omega^4 \end{gathered}}$$

$k_1k_2 + \cancel{k_2^2} - m_1k_2\omega^2 - k_1m_2\omega^2 - k_2m_2\omega^2 + m_1m_2\omega^4 - \cancel{k_2^2}$

$(m_1m_2)\omega^4 - (m_1k_2 + k_1m_2 + k_2m_2)\omega^2 + k_1k_2 = 0$

$(m_1m_2)(\omega^2)^2 - [k_1m_2 + (m_1+m_2)k_2]\omega^2 + k_1k_2 = 0$

$$\omega_{1,2}^2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$1.2(\omega^2)^2 - [\cancel{40}\times 32 + (92)](\omega^2)' + 1600 = 0$

$1.2(\omega^2)^2 - 124(\omega^2)' + 1600$

$$\frac{124 \pm \sqrt{15376-7680}}{2.4}$$

$$\omega_{1,2}^2 = \frac{124 \pm 87.72}{2.4} = \omega$$

$$\boxed{\begin{gathered} \omega_{1,2}^2 = 88.21 \\ \omega_{1,2}^2 = 15.11 \end{gathered}}$$

$\omega_1 = 9.39$ rad/sec

$\omega_2 = 3.88$ rad/sec

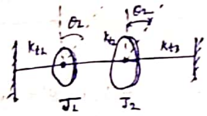
$$\left(\frac{A_1}{A_2}\right)_1 = \frac{40}{40+40-1.5\times(9.39)^2} = -0.76$$

$$\left(\frac{A_1}{A_2}\right)_2 = \frac{40-0.8\times(3.88)^2}{40} = 0.69 \text{ Ans}$$

Q2.

$J_1 = J_0$, $J_2 = 2J_0$

$k_{t1} = k_{t2} = k_{t3}$.



for man moment of inertia 1.

~~$J_1\ddot{\theta}_1 + k_{t1}(\theta_1 + k_{t2}\theta_1) + k_{t3}\theta_2$~~

$J_1\ddot{\theta}_1 + k_{t1}\theta_1 + k_{t2}(\theta_1-\theta_2) = 0$ —— ①

→ $J_1\ddot{\theta}_1 + k_{t1}\theta_1 + k_{t2}\theta_1 - k_{t2}\theta_2 = 0$

$J_1\ddot{\theta}_1 + k_{t1}(\theta_1-\theta_2) + k_{t2}\theta_1 =$

for man moment of inertia ②.

$J_2\ddot{\theta}_2 + k_{t2}(\theta_2-\theta_1) + k_{t3}\theta_2 = 0$ —— ②

~~$J_2\ddot{\theta}_2 + k_{t2}\theta_2 - k_{t2}\theta_2$~~

$J_2\ddot{\theta}_2 - k_{t2}\theta_1 + ~~k_{t2}(\theta_1+\theta_2)~~ (k_{t3}+k_{t2})\theta_2$ — ②

$\theta_1 = A_1\sin\omega t$, $\theta_2 = A_2\sin\omega t$

$\ddot{\theta}_1 = -A_1\omega^2\sin\omega t$, $\ddot{\theta}_2 = -A_2\omega^2\sin\omega t$

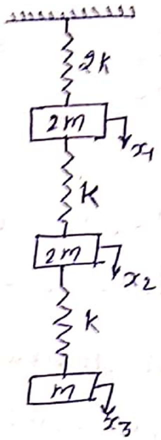
$-J_1A_1\omega^2 + k_{t1}(A_1-A_2) + k_{t2}A_1 = 0$

$= -J_1A_1\omega^2 + k_{t1}A_1 - k_{t1}A_2 + k_{t2}A_1 = 0$

$A_1(k_{t1}+k_{t2}-J_1\omega^2) - k_{t1}A_2$

$$\boxed{\left(\frac{A_1}{A_2}\right)_1 = \frac{k_{t1}}{k_{t1}+k_{t2}-J_1\omega^2}}$$

UNIT-VIth many degree of freedom System:-



Eqn of motion. for mass m_1

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = 0$$

$$m_1\ddot{x}_1 + x_1(k_1+k_2) - k_2x_2 = 0 \quad ①$$

from mass m_2

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_3(x_2 - x_3) = 0$$

$$m_2\ddot{x}_2 + x_2(k_2+k_3) - k_2x_1 - k_3x_3 = 0 \quad ②$$

for mass m_3.

$$m_3\ddot{x}_3 + k_3(x_3 - x_2) = 0$$

$$m_3\ddot{x}_3 \cancel{+} k_3x_2 + k_3x_3 = 0 \quad ③$$

$$x_1 = A_1\sin\omega t \quad , \quad \ddot{x}_1 = -A_1\omega^2\sin\omega t$$

$$x_2 = A_2\sin\omega t \quad , \quad \ddot{x}_2 = -A_2\omega^2\sin\omega t$$

$$x_3 = A_3\sin\omega t \quad , \quad \ddot{x}_3 = -A_3\omega^2\sin\omega t$$

from equation ①.

$$-m_1A_1\omega^2 + A_1k_1 + k_2(A_1 - A_2) = 0$$

$$-m_1A_1\omega^2 + A_1k_1 + k_2A_1 - k_2A_2 = 0$$

$$A_1(k_1 + k_2 - m_1\omega^2) = k_2A_2 \quad ④$$

= from matrix method:-

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = 0$$

$$[M]\{\ddot{x}\} + [K][x] = 0$$

[EigenValue & Eigen Vector]

frequency $\begin{cases} \omega_1 \\ \omega_2 \\ \omega_3 \end{cases}$ $\quad$ $\left(\frac{A_1}{A_2}\right)_1$ —

$\frac{A_1}{A_2}$

$$[M]^{-1}[M][\ddot{x}] + [M]^{-1}[K][x] = 0$$

Identity matrix → $[I][\ddot{x}] + [m]^{-1}[K]\{x\} = 0$

↑ Dynamic matrix

$\{x\} = \{A\}\sin\omega t$

$\{\ddot{x}\} = \{-\omega^2\}A\sin\omega t$

$\{\ddot{x}\} = \{-\omega^2\}\{x\}.$

$\ddot{x} = -\lambda\{x\}$

$$\boxed{[[D] - \lambda[I]]\{x\} = 0}$$

$[m^{-1}] = \frac{adj[m]}{|m|}$

for mass m_1

$2m\ddot{x}_1 + 3Kx_1 - Kx_2 = 0$ —— ①

$2m\ddot{x}_2 - Kx_1 + 2Kx_2 - Kx_3 = 0$ —— ②

$m\ddot{x}_3 - Kx_2 + Kx_3 = 0$ —— ③

$$\begin{bmatrix} 2m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & 2m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 3K & -K & 0 \\ -K & 2K & -K \\ 0 & -K & K \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$[m][\ddot{x}] + [K][x] = 0$

$$\boxed{[D] - \lambda[I] = 0}$$

$[m]^{-1} = \frac{adj[m]}{|m|}$

$|M| = 2m(2m^2 - 0)$

$= 4m^2$

$a_{11} = 2m^2 \quad a_{12} = 0 \quad a_{13} = 0$

$a_{21} = 0 \quad a_{22} = 2m^2 \quad a_{23} = 0$

$a_{31} = 0 \quad a_{32} = 0 \quad a_{33} = 4m^2$

$$\begin{bmatrix} 2m^2 & 0 & 0 \\ 0 & 2m^2 & 0 \\ 0 & 0 & 4m^2 \end{bmatrix}^T = \begin{bmatrix} 2m^2 & 0 & 0 \\ 0 & 2m^2 & 0 \\ 0 & 0 & 4m^2 \end{bmatrix}$$

$$[m]^{-1} = \frac{1}{4m^3}\begin{bmatrix} 2m^2 & 0 & 0 \\ 0 & 2m^2 & 0 \\ 0 & 0 & 4m^2 \end{bmatrix}$$

$$[m]^{-1} = \begin{bmatrix} \frac{1}{2m} & 0 & 0 \\ 0 & 1/2m & 0 \\ 0 & 0 & 1/2m \end{bmatrix}$$

$$[M]^{-1}[K] = \begin{bmatrix} \frac{1}{2m} & 0 & 0 \\ 0 & \frac{1}{2m} & 0 \\ 0 & 0 & \frac{1}{m} \end{bmatrix}\begin{bmatrix} 3K & -K & 0 \\ -K & 2K & -K \\ 0 & -K & K \end{bmatrix}$$

$$[m]^{-1}[K] = \begin{bmatrix} 3K/2m & -K/2m & 0 \\ -K/2m & K/m & -K/2m \\ 0 & -K/m & K/m \end{bmatrix}$$

$[m]^{-1}[K] - \lambda[I] = \begin{bmatrix} 3K/2m - \lambda & -K/2m - \lambda & 0\lambda \\ -K/2m - \lambda \end{bmatrix}$

$$[M]^{-1}[K]-\lambda[I]=\begin{bmatrix}\frac{3K}{2m} & -\frac{K}{2m} & 0\\ -K/2m & K/m & -K/2m\\ 0 & -K/m & K/m\end{bmatrix}-\begin{bmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix}$$

$$=\begin{bmatrix}\frac{3K}{2m}-\lambda & -K/2m & 0\\ -K/2m & K/m-\lambda & -K/2m\\ 0 & -K/m & K/m-\lambda\end{bmatrix}$$

$$\left(\frac{3K}{2m}-\lambda\right)\left[\left(K/m-\lambda\right)^2-\frac{K^2}{2m^2}\right]+\frac{K}{2m}\left[-K/2m\left(K/m-\lambda\right)\right]$$

$$\left(\frac{K}{m}-\lambda\right)\left[\left\{\left(\frac{3}{2}-\lambda\right)\left[\left(K/m-\lambda\right)^2-\frac{K^2}{2m^2}\right]\right\}-\frac{K^2}{4m^2}\right]$$

$$\left(\frac{3K}{2m}-\lambda\right)\left[\frac{K^2}{m^2}-\frac{2K}{m}\lambda+\lambda^2-\frac{K^2}{2m^2}\right]-\frac{K^2}{4m}\left(K/m-\lambda\right)\Big]$$

$$\left(\frac{3K}{2m}-\lambda\right)\left[\frac{K^2}{m^2}-\frac{2K}{m}\lambda+\lambda^2-\frac{K^2}{2m^2}\right]-\left(\frac{-K^2}{4m}\right)\left[\frac{K}{m}-\lambda\right]$$

$$\cancel{\frac{3K^3}{2m^3}-\frac{6K^2}{2m^2}\lambda+\frac{3K}{2m}\lambda^2-\frac{3K^3}{2m^3}-\frac{K^2}{m^2}\lambda}$$

$$\left[\frac{3K^3}{2m^3}-\frac{K^2}{m^2}\lambda-\frac{6K^2}{2m^2}\lambda+\frac{2K}{m}\lambda^2+\frac{3K}{2m}\lambda^2-\lambda^3-\frac{3K^3}{2m^3}+\frac{K^2}{2m^2}\lambda\right]$$

$$-\left[\frac{K^3}{4m^2}-\frac{K^2}{4m}\lambda\right]$$

$$\frac{3K^3}{2m^3}-\frac{K^2}{m^2}\lambda-\frac{6K^2}{2m^2}\lambda+\frac{2K}{m}\lambda^2+\frac{3K}{2m}\lambda^2-\lambda^3-\frac{3K^3}{2m^3}+\frac{K^2}{2m^2}\lambda-\frac{K^3}{4m^3}+\frac{K^2}{4m^2}\lambda$$

$$-\lambda^3+\lambda^2\left[\frac{2K}{m}+\frac{3K}{2m}\right]-\lambda\left[\frac{K^2}{m^2}+\frac{6K^2}{2m^2}-\frac{K^2}{2m^2}-\frac{K^2}{4m^2}\right]+$$

$$\cancel{\frac{3K^3}{2m^3}}-\cancel{\frac{3K^3}{2m^3}}+\frac{K^3}{4m^3}$$

$$-\lambda^3+\lambda^2\left[\frac{7K}{2m}\right]-\lambda\left[\frac{4K^2+12K^2-2K^2-K^2}{4m^2}\right]+\frac{K^3}{4m^3}$$

$$\boxed{\lambda^3-\frac{7K}{2m}\lambda^2+\frac{13}{4}\frac{K^2}{m^2}\lambda-\frac{K^3}{4m^3}=0}$$

$$\boxed{y^3+py^2+qy+r=0}$$

$\hookrightarrow$ $y_1=g\cos\phi/3-p/3$, $y_2=g\cos(\phi/3+120°)-p/3$

$y_3=g\cos(\phi/3-240°)-p/3$

$g=2\sqrt{-q/3}$, $\cos\phi=\dfrac{b}{2\left(\frac{-q^3}{27}\right)^{1/2}}$, $q=1/3(3q-p^2)$

$b=1/27(2p^3-9pq+27r)$

$$q=1/3\left[3\times\frac{13}{4}\times\frac{K^2}{m^2}-\left(\frac{7K}{2m}\right)^2\right]$$

$$=\frac{1}{3}\left[\frac{39}{4}\frac{K^2}{m^2}-\frac{49}{4}\times\frac{K^2}{m^2}\right]\qquad\boxed{q=-\frac{5}{6}\frac{K^2}{m^2}}$$

~~$b=\frac{1}{27}(2\times$~~ $g=2\sqrt{\frac{5\times K^2}{6\times3\,m^2}}$ $\quad g=\frac{2K}{m}\sqrt{\frac{5}{18}}\Rightarrow 1.05\frac{K}{m}$

$$b = \frac{1}{27}\left[2 \times \frac{343}{6m^3} k^3 + 1.05\frac{k}{m} \times \frac{7k}{2m} \times \frac{13}{4} \frac{k^2}{m^2} + 27 \times \frac{1}{4} \frac{k^3}{m^3}\right]$$

$$\frac{1}{27}\left[85.75\frac{k^3}{m^3} + 11.37\frac{k^3}{m^3} - 6.75\, k^3/m^3\right]$$

$$b = \frac{90.37}{27} \frac{k^3}{m^3}$$

$$\boxed{b = 3.34 \frac{k^3}{m^3}}$$ ✓

$y_1 =$ [illegible] $\cos\phi = \dfrac{b}{2\left(\frac{-a^3}{27}\right)^{1/2}} = \dfrac{3.34 \frac{k^3}{m^3}}{2 \times \left(\frac{15}{26 \times 27} \times \frac{k^6}{m^6}\right)^{1/2}}$

$$\cos\phi = \frac{3.34\, k^3/m^3}{2 \times 0.146 \left(\frac{k^{6 \times 1/2}}{m^{6 \times 1/2}}\right)^{1/2}}$$

✓

$$\boxed{\cos\phi = 11.43}$$

$$\phi = \cos^{-1}(11.43)$$

$$\boxed{\cancel{\frac{\cos\phi}{12} = \frac{11.43}{12}}}$$